Interpretation of Duality and Duality theorem

Dualization for everyone:
$A \in \mathbb{R}^{m\times n}, \mathbf{c} \in \mathbb{R}^n, \mathbf{b} \in \mathbb{R}^m$

	primal	dual
variables	$x_1, \dots, x_n$	$y_1, \dots, y_m$
matrix	A	A^T
right hand	$\mathbf{b}$	$\mathbf{c}$
objective	max $\mathbf{c}^T\mathbf{x}$	min $\mathbf{b}^T\mathbf{y}$
constraint	ith constrain $\leq$	$y_i \geq 0$
	ith constrain $\geq$	$y_i \leq 0$
	ith constrain $=$	$y_i \in \mathbb{R}$
	$x_i \geq 0$	ith constrain $\geq$
	$x_i \leq 0$	ith constrain $\leq$
	$x_i \in \mathbb{R}$	ith constrain $=$

Diet problem: How much apricots (x_1), bananas (x_2) and cucumbers (x_3) does one have to eat to get enough of Vit A, C, K? Minimize the cost.

Need to know: % of daily value and cost:

	A	C	K	\$	ammount
apricots	60	26	6	1.53	155g
bananas	3	33	1	0.37	225g
cucumbers	2	7	12	0.18	133g

1: Write Linear Program (P) solving the diet problem and write also its dual (D).

2: What are units of y_i in (D)?
(Hint: inequalities need to make sense in units. This is known as a *dimensional analysis.*)
(y_is are known as *shadow prices.*)

3: Imagine you want to create a multivitamine pills ACK. What is the maximum price of one *ACK* pill if it has to deliver 100% of recommended daily value of vitamins A,C, and K and it must beat any fruit and vegetable in terms of price?
(If you don't manage to beat fruit and vegetable, nobody will buy your fancy ACK pill.)
(Compute your price as a combination of prices of each of the vitamins.)
(Look at (D).)

Strong Duality Theorem
For the linear programs

$$\text{maximize } \mathbf{c}^T\mathbf{x} \text{ subject to } A\mathbf{x} \leq \mathbf{b} \text{ and } \mathbf{x} \geq \mathbf{0} \qquad (P)$$

and

$$\text{minimize } \mathbf{b}^T\mathbf{y} \text{ subject to } A^T\mathbf{y} \geq \mathbf{c} \text{ and } \mathbf{y} \geq \mathbf{0} \qquad (D)$$

exactly one of the following possibilities occurs:

1. Neither (P) nor (D) has a feasible solution.
2. (P) is unbounded and (D) has no feasible solution.
3. (P) has no feasible solution and (D) is unbounded.
4. Both (P) and (D) have a feasible solution. Then both have an optimal solution, and if $\mathbf{x}^\star$ is an optimal solution of (P) and $\mathbf{y}^\star$ is an optimal solution of (D), then

   $$\mathbf{c}^T\mathbf{x}^\star = \mathbf{b}^T\mathbf{y}^\star.$$

   That is, *the maximum of* (P) *equals the minimum of* (D).